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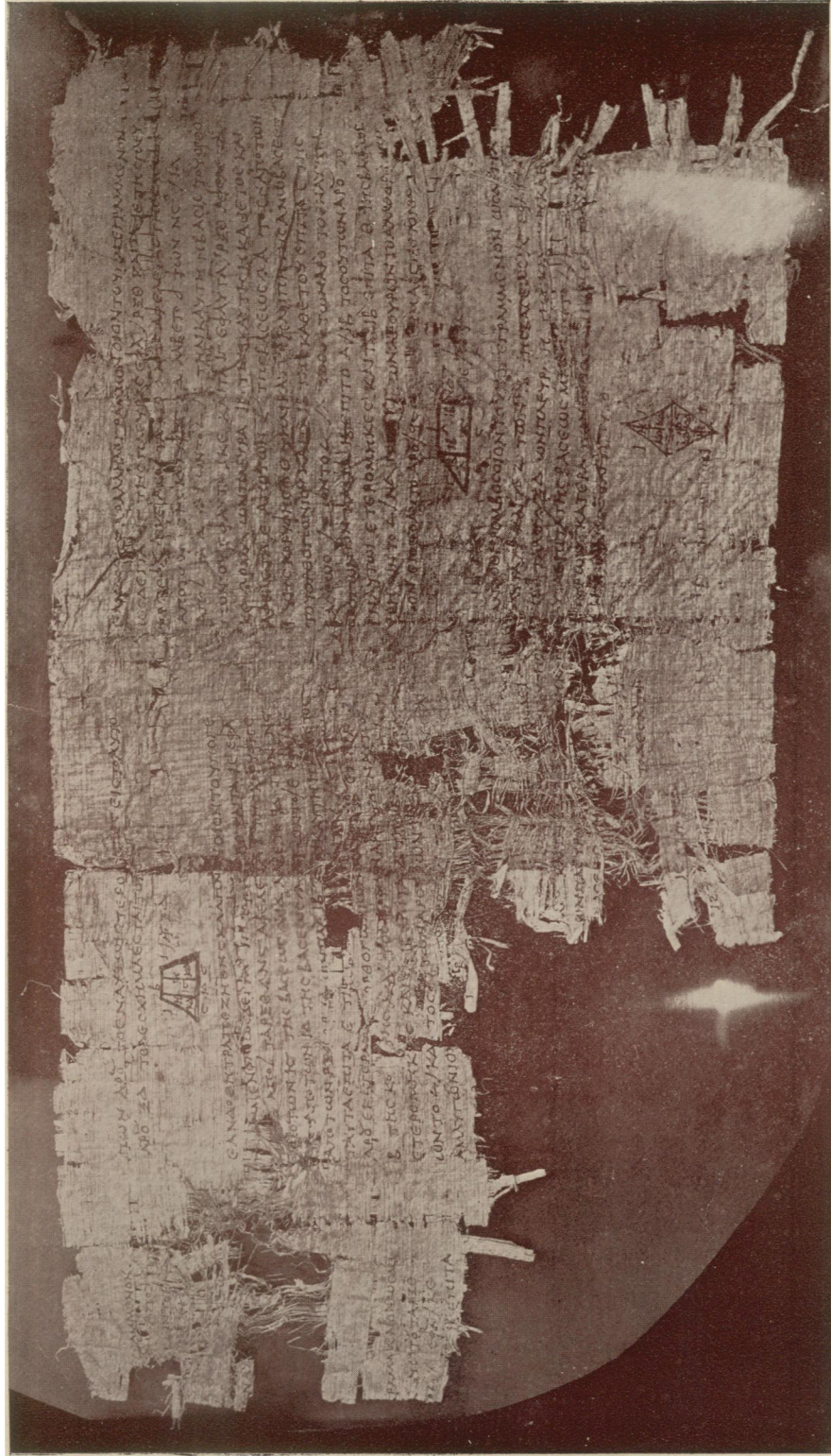
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THE AYER PAPYRUS.

Photographed by Allen, Field Columbian Museum.

II.—THE AYER PAPYRUS: A MATHEMATICAL FRAGMENT.

About three years ago, Mr. Ed. E. Ayer, of Chicago, observed a Greek papyrus of unusual aspect in a small shop in Cairo. The fragment was in clear uncial interspersed with geometrical figures; and its evident mathematical import so interested Mr. Ayer that he purchased it. After leaving it for a year in the Ghizeh Museum, Mr. Ayer brought it to Chicago, where it now lies in the Egyptian Room of the Field Columbian Museum, of the trustees of which Mr. Ayer is president. Mr. Ayer mentioned the papyrus to Dr. James Henry Breasted, of the University of Chicago, who brought it to my notice. It is thus through the kindness of Dr. Breasted that I am able to present this statement as to the fragment. The only precise information obtainable as to the place where the papyrus was found comes from Mr. Grenfell, of Oxford. Mr. Grenfell kindly writes me that he saw the fragment in Egypt some four years ago, in the hands of a dealer who said that he found it at Hawara, in the Fayum, near the pyramid; and as he was known to have been digging there, his story may have been true. This accords with what was said to Mr. Ayer, at the time the fragment was purchased.

For valuable suggestions and counsel in my work on the papyrus, I am under obligations to Professor Beman, of the University of Michigan; Professor Mahaffy, of Dublin, and Dr. Fr. Krebs, of Berlin. No one of these, however, is to be held responsible for any of the views presented in this article. Through the kindness of Professor Beman, I am able also to present suggestions from M. Tannery, the editor of *Diophantus*; Dr. Hultsch, of Dresden, the editor of *Heron*; and M. Heiberg, of Copenhagen. While these suggestions are based upon an examination of a provisional transcription of the first process only, they have proved of great value.

The fragment contains about thirty-five complete lines, with parts of perhaps half as many more. Originally it formed part of a papyrus roll, inscribed in clear uncials on one side only, the writing being in columns slightly longer than broad. Portions of

three columns remain. In shape the fragment is irregular, the first column being almost entirely gone, while the second lacks its closing lines, and the third is practically complete. The extreme dimensions of the papyrus are cm. 21.3 by 40.5. The third column, the only complete one, measures cm. 17.5 in height, including the figure of a rhomb at its foot, and 14.5 in breadth. Of column I there remain parts of the closing words of eleven lines. Column II preserves twenty-four lines, some of them much mutilated, with one figure, a trapezoid. Column III consists of twenty-one lines and two figures, a parallelogram and a rhomb. From what remains it can readily be seen that the three columns contained seven processes, of which the papyrus gives us inconsiderable fragments of the first, second and fifth, the two concluding lines and the figure of the third, the text but not the figure of the fourth, and both text and figures of the sixth and seventh. The geometrical figures illustrating the processes described are appended to the processes, and are covered with numerals indicative of the length of each side, part of a side, and perpendicular, and the area of each section. They are accompanied also by short sentences giving the final result of the calculation; as, e. g., $\gamma \text{ ἀποὺ } \Xi\Delta$, after Fig. 1. Occasional lacunae in the text of 4, 6 and 7 are readily filled from the context, or the parallels afforded by other portions, such restoration being facilitated by the uniformity of the language used. The letters are mm. 2 to 2.5 high and run 35 to 39 to the line. Iota does not appear as subscript; it is adscript in $\alpha\upsilon\tau\tau\hat{\omega}$, but is not written in the subjunctive $\delta\omicron\theta\hat{\eta}$. Breathings, accents and punctuation are wanting, and the letters are seldom separated into words. Abbreviations and symbols play a prominent part in the processes.

In the absence of more positive information regarding the finding of the papyrus and the date of any commercial or official material that may have been found with it, it is hazardous to attempt to fix its date. That the work was a roll precludes a date later than the early fourth century for the manuscript; and the clear, free and unexaggerated literary uncials, devoid of points and accents, and rarely separated into words, seem to require a date not later than 200 A. D. Dr. Krebs favors the second century, or possibly the third. Professor Mahaffy inclines toward the latter part of the first; and Mr. Grenfell informs me that in the Oxyrhynchus collection there are several similar papyri dated in the reigns of Vespasian and Trajan. The early

second century would thus seem to be a conservative date for the fragment.

One or two things confirm the natural presumption that the fragment was a copy, and suggest that it was a somewhat careless one. Such peculiarities as the writing of *τραπέζηον* for *τραπέζειον* or *τραπέζιον*, and of *ἀμλνγώνιον* for *ἀμβλνγώνιον*, may be otherwise explained. But the writing of *ὀρογώνιον* for *ὀρθογώνιον*, col. III 12, and the unmistakable displacement of *ἡμισυ*, col. II 7—where we must read *λοιπὰ Ἰ ὧν τὸ ἡμισυ γίνεται Ε κτέ.*—are clearly scribal errors; and in examining the so-called parallelogram of col. III, one is struck with the fact that while the sides of the figure are 6, 13, 10, 15, the scribe has drawn a Euclidean parallelogram with opposite sides equal, and then, apparently oblivious of the incongruity, has given to its sides the values prescribed in his exemplar. Clearly the scribe was not a convert to that geometrical school which required of a parallelogram only that two sides of it be parallel. In the other figures the scribe has been equally careless. No particular effort seems to have been made to draw them to scale. In the first one, the evident intention of the perpendicular is to cut the figure into two triangles and a rectangle; but without the numbers that rescue it, the figure would suggest anything but this. The scribe was perhaps absent-minded, and unconsciously exaggerated the inaccuracies of his copy in the figures, while writing in the correct numbers. In the papyrus the trapezoid measures cm. 1.1 by 2; the parallelogram, cm. 1 by 3.8; the rhomb, 2.8 by 1.6.

In the following transcriptions the fragments of column I and of the last process of column II are not presented; and in the index, the scanty material which they afford is not included. Their contribution to the matter of abbreviations is treated with the other abbreviations of the manuscript.

As to the nature and purpose of the book of which we have a fragment, it has been suggested by Professor Mahaffy and others that it was a practical treatise on mensuration, designed for use in resurveying farm-lands of irregular shape, after inundations, etc. The word *ἄρουρα*, however, does not seem to have quite the meaning it ordinarily had in Egypt, viz. 'acre,' but is employed here simply as a unit of square measure. Both M. Tannery and Dr. Hultsch have suggested that *ἄρουρα*, in its practical, specific sense at least, was employed under the Ptolemies, but seems to have gone out of use before the time of Heron Alexandrinus,

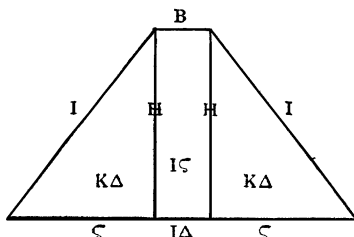
who reckons areas by *σχοινία*. I have observed *ἄρουρα* in its specific sense, in a number of papyri of the second century A. D., but its absence from Heron is certainly significant. This, taken with what seems the archaic use of *παραλληλόγραμμον*, may yet carry the date of the origin of the work into pre-Christian times. A few deviations from the familiar syntax will be observed, and perhaps these may, under the hands of scholars familiar with Greek of the period, contribute to more exact determination of date and origin. As M. Tannery and Dr. Hultsch have pointed out, we should expect the singular *ἐκάτερον* in place of the plural *ἐκάτερα* in the first process, line 8. Dr. Hultsch further calls attention to the fact that the side-lengths are carefully chosen with a view to having the square roots required come out whole numbers.

In the following transcription, all abbreviations have been expanded, such expansions being indicated by parentheses (). Restorations are enclosed in brackets; expanded symbols are in lower-case. The figure following process 2 is a restoration suggested in substantially the same form, by Professor E. H. Moore, Dr. Hultsch and M. Heiberg. The first lines of the third process of col. I may be supplied about as follows, on the basis of the figure near the top of col. II:—

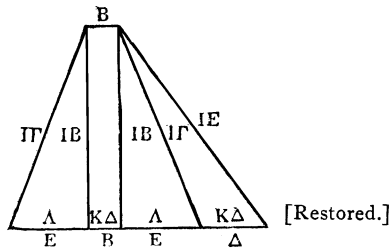
[ἐὰν δοθῇ τραπέζιον ἰσοσκελές οἷον τὸ ὑπογεγραμμένον ὥς δεῖ τὰ Ι ἐφ' αὐτὰ γίνεται Ρ ἄφελε τὰ Β τῆς κορυφῆς ἀπὸ τῶν ΙΔ τῆς βάσεως λοιπὰ ΙΒ ὧν τὸ ἥμισυ γίνεται Σ ἐφ' αὐτὰ γίνεται ΛΣ ἄφελε τὰ ΛΣ λοιπὰ ΞΔ ὧν πλευρὰ Η τηλικαύτῃ ἢ κάθετος ὧν τὸ ἥμισυ γίνεται Δ ταῦτα ἐπὶ τὰ Σ τῆς βάσεως γίνεται ΚΔ τοσούτων ἀρουρῶν ἐκάτερα τῶν ὀρθογωνίων καὶ τὰ Η τῆς καθέτου ἐπὶ τὰ Β τῆς βάσεως γίνεται ΙΣ τοσού-]

Column II:

ΤΩΝ ΑΡΟΥ(ΡΩΝ) ΤΟ ΕΝ ΑΥΤΩΙ ΕΤΕΡΟΜΗΚΕΣ ΕΙΣ ΤΟ ΑΥΤΟ
ΑΡΟΥ(ΡΑΙ) ΞΔ ΤΟ ΔΕ ΣΧΗΜΑ ΕΣΤΑΙ ΤΟΙΟΥΤΟ



ΕΑΝ ΔΟΘΗ ΤΡΑΠΕΖΗΟΝ ΣΚΑΛΗΝΟΝ ΟΙΟΝ ΤΟ ΥΠΟΓΕ
 ΓΡΑΜ]ΜΕΝΟΝ ΩΣ ΔΕΙ ΤΑ ΙΓ ΕΦ ΑΥ(ΤΑ) [γίνεται Ρ]ΞΘ ΚΑΙ ΤΑ ΙΕ ΕΦ
 ΑΥ(ΤΑ)
 γίνεται ΣΚ]Ε ΑΠΟ τούτων ΤΑ ΡΞΘ λοιπὰ Νς ΑΦΕΛΕ ΤΑ Β ΤΗΣ ΚΟΡΥΦΗΣ
 ΑΠΟ ΤΩΝ Ις ΤΗΣ ΒΑΣΕΩΣ λοιπὰ ΙΔ ΛΑΒΕ ΤΟ Ι'Δ' ΤΩΝ Νς
 γίνεται Δ ΑΠΟ ΤΩΝ ΙΔ ΤΗΣ ΒΑΣΕΩΣ λοιπὰ ἤμισυ Ι ΩΝ ΤΟ γίνεται Ε ΕΦ
 ΑΥ(ΤΑ) γίνεται ΚΕ
 ΑΠΟ ΤΩΝ ΡΞΘ λοιπὰ ΡΜΔ ΩΝ ΠΛΕΥ(ΡΑ) ΙΒ ΘΛΙΚΑΥΤΗ Η ΚΑΘΕΤΟΣ
 ΤΑΥΤΑ ΕΠΙ ΤΑ Ε ΤΗΣ ΒΑ[ΣΕΩΣ γίνεται Ξ] ΩΝ ΤΟ ἤμισυ γίνεται Λ
 ΤΟΣΟΥΤΩΝ
 ΑΡΟΥ(ΡΩΝ) ΕΚΑΤΕΡΑ Τ[ΩΝ] ΟΡΘΟΓΩΝΙΩΝ ΚΑΙ ΤΑ ΙΒ ΕΠΙ ΤΑ
 Β ΤΗΣ ΚΟ[ΡΥ]ΦΗΣ γίνεται ΚΔ ΤΟΣ[Ο]ΥΤΩΝ ΑΡΟΥ(ΡΩΝ) ΤΟ ΕΝ ΑΥΤΩΙ
 ΕΤΕΡΟΜΗΚΕΣ ΚΑΙ ΤΑ ΙΒ ΕΠΙ ΤΑ Δ ΤΗΣ ΒΑΣ[ΕΩ]ς γίνεται ΜΗ
 ΩΝ ΤΟ ἤμισυ γίνεται ΚΔ ΤΟΣΟΥΤΩΝ ΑΡΟΥΡΩΝ ΤΟ ΕΝ [ΑΥ]ΤΩΙ
 ΑΜΑΥΓΩΝΙΟΝ [ΕΙΣ ΤΟ Α]ΥΤΟ γίνονται ΑΡΟΥ(ΡΑΙ) ΡΗ ΤΟ [ΔΕ ΣΧΗΜ]Α
 ΕΣΤΑΙ
 ΤΟΙΟΥΤ]Ο

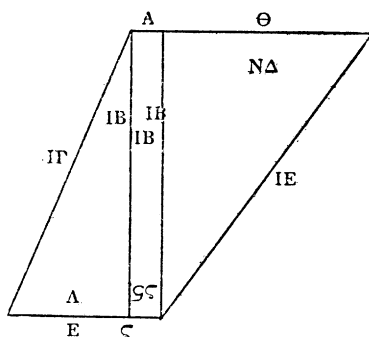


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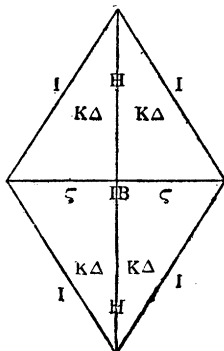
Column III :

ΕΑΝ ΔΟΘΗ ΠΑΡΑΛΛΗΛΟΓΡΑΜΜΟΝ ΟΙΟΝ ΤΟ ΥΠΟΓΕΓΡΑΜΜΕΝΟΝ
 ΩΣ ΔΕΙ ΤΑ ΙΓ ΤΗΣ ΠΛΕΥΡΑΣ ΕΦ ΑΥ(ΤΑ) γίνεται ΡΞΘ ΚΑΙ ΤΑ ΙΕ ΤΗΣ
 ΠΛΕΥ
 ΡΑΣ ΕΦ ΑΥ(ΤΑ) γίνεται ΣΚΕ ΑΠΟ τούτων ΤΑ ΡΞΘ λοιπὰ Ν[ς] ΑΦΕΛΕ ΤΑ
 ς ΤΗΣ ΒΑΣΕΩΣ
 ΑΠΟ ΤΩΝ Ι ΤΗΣ ΚΟΡΥΦΗΣ λοιπὰ Δ ΛΑΒΕ ΤΟ τέταρτον ΤΩΝ Νς γίνεται ΙΔ
 ΑΠΟ τούτων ΤΑ Δ λοιπὰ Ι ΩΝ ΤΟ ἤμισυ γίνεται Ε ΘΛΙΚΑΥΤΗ Η ΒΑΣΙΣ
 ΤΟΥ ΟΡΘΟ
 ΓΩΝΙΟΥ ΕΦ ΑΥΤΑ γίνεται ΚΕ ΚΑΙ ΤΑ ΙΓ ΕΦ ΑΥΤΑ γίνεται ΡΞΘ ΑΦΕΛΕ ΤΑ
 ΚΕ λοιπὰ ΡΜΔ ΩΝ ΠΛΕΥΡΑ ΙΒ ΘΛΙΚΑΥΤΗ Η ΚΑΘΕΤΟΣ ΚΑΙ
 ΑΦΕΛΕ ΤΑ Ε ΑΠΟ ΤΩΝ ς ΤΗΣ ΒΑΣΕΩΣ λοιπὸν Α ΤΟ ΕΝ ΑΠΟ ΤΩΝ

Ι ΤΗΣ ΚΟΡΥΦΗΣ λοιπὰ Θ ΤΗΛΙΚΑΥΤΗ Η ΛΟΙΠΗ ΤΗΣ ΑΝΩ ΒΑΣΕΩΣ
 ΤΟΥ ΟΡΘΟΓΩΝΙΟΥ ΚΑΙ ΤΑ ΙΒ ΤΗΣ ΚΑΘΕΤΟΥ ΕΠΙ ΤΑ Ε ΤΗΣ
 ΒΑΣΕΩΣ γίνεται Ξ ΩΝ ΤΟ ἥμισυ γίνεται Λ ΤΟΣΟΥΤΩΝ ΑΡΟΥ(ΡΩΝ) ΤΟ ΕΝ
 ΑΥΤ[Ω]Ι
 ΟΡΘΟΓΩΝΙΟΝ ΚΑΙ ΤΑ ΙΒ ΕΠΙ ΤΟ Α γίνεται ΙΒ ΤΟΣΟΥΤΩΝ ΑΡΟΥ(ΡΩΝ) ΤΟ
 ΕΝ ΑΥΤΩΙ ΕΤΕΡΟΜΗΚΕΣ ΚΑΙ ΤΑ ΙΒ ΕΠΙ ΤΑ Θ ΤΗΣ ΒΑΣΕΩΣ
 γίνεται ΡΗ ΩΝ ΤΟ ἥμισυ γίνεται ΝΔ ΤΟ[ΣΟΥΤ]ΩΝ ΑΡΟΥΡΩΝ ΤΟ ΑΛΛΟ
 ΟΡΘΟΓΩ
 ΝΙΟΝ ΕΙΣ ΤΟ ΑΥΤΟ ΑΡΟΥ(ΡΑΙ) γίνονται ςς ΤΟ ΔΕ ΣΧΗΜΑ ΕΣΤΑΙ
 ΤΟΙΟΥ[ΤΟ



ΕΑΝ ΔΟΘΗ ΡΟΜΒΟΣ ΟΙΟΝ ΤΟ ΥΠΟΓΕΓΡΑΜΜΕΝΟΝ ΩΣ ΔΕΙ ΤΑ Ι
 Ε]Φ ΑΥΤΑ γίνεται Ρ ΚΑΙ ΤΟ ἥμισυ ΤΩΝ ΙΒ ΤΗΣ ΒΑΣΕΩΣ γίνεται ς ΕΦ
 ΑΥΤΑ γίνεται Λς
 ΑΠΟ τούτων ΤΑ Λς λοιπὰ ΞΔ ΩΝ ΠΛΕΥΡΑ Η ΤΗΛΙΚΑΥΤΗ Η ΚΑΘΕΤΟΣ
 ΤΑΥΤΑ ΕΠΙ ΤΑ ΤΗΣ ΒΑΣΕΩΣ Μ[Η] ΩΝ ΤΟ ἥμισυ γίνεται Κ[Δ] ΤΟΣΟΥΤΩΝ
 ΑΡ]ΟΥΡΩΝ ΕΚΑΤΕΡΑ ΤΩΝ ΟΡΘΟΓΩΝΙΩΝ ΕΙΣ ΤΟ ΑΥ[ΤΟ ΑΡΟΥ(ΡΑΙ) ςς
 ΤΟ ΔΕ Σ]Χ[ΗΜΑ Ε]ΣΤΑΙ ΤΟΙΟΥΤΟ



In the appended translation, superior numerals refer to lines of the columns in the papyrus.

Column II :

¹ Of so many units is the rectangle in it. Altogether ² 64 units. And the figure will be as follows.

³ If there be given a scalene trapezoid such as the one drawn below ⁴ according to the conditions of the problem the ¹³ squared is 169 and the ¹⁵ squared ⁵ is 225. Subtract 169. 56 remains. Subtract the 2 of the upper side ⁶ from the 16 of the base. 14 remains. Take $\frac{1}{4}$ of the 56. ⁷ It is 4. From the 14 of the base; there remains 10, of which $\frac{1}{2}$ is 5. (This) squared is 25. ⁸ (Take this) from the 169; 144 remains, of which the square root (is) 12. So great is the perpendicular. ⁹ This (multiplied) by the 5 of the base is 60, of which $\frac{1}{2}$ is 30. Of so many ¹⁰ units is each of the right-angled triangles. And the 12 by the ¹¹ 2 of the upper side is 24. Of so many units is the ¹² parallelogram in it. And the 12 by the 4 of the base is 48, ¹³ $\frac{1}{2}$ of which is 24. Of so many units is the ¹⁴ obtuse-angled triangle in it. Altogether it is 108 units. And the figure will be ¹⁵ as follows.

Column III :

¹ If there be given a parallelogram such as the one drawn below ² according to the conditions of the problem the ¹³ of the side squared is 169 and the ¹⁵ of the side ³ squared is 225. Subtract the 169. 56 remains. Subtract the 6 of the base ⁴ from the 10 of the upper side. 4 remains. Take $\frac{1}{4}$ of the 56. It is 14. ⁵ Subtract the 4. 10 remains, $\frac{1}{2}$ of which is 5. So great is the base of the right-⁶angled triangle. (This) squared is 25. And the ¹³ squared is 169. Subtract the ⁷ 25. 144 remains, the square root of which is 12. So great is the perpendicular. And ⁸ subtract the 5 from the 6 of the base. 1 remains. (Take) the one from the ⁹ 10 of the upper side. 9 remains. So great is the remainder of the upper base ¹⁰ of the right-angled triangle. And the 12 of the perpendicular by the 5 of the ¹¹ base is 60, $\frac{1}{2}$ of which is 30. Of so many units is the ¹² right-angled triangle in it. And the 12 by the 1 is 12. Of so many units is the ¹³ rectangle in it. And the 12 by the 9 of the base ¹⁴ is 108, $\frac{1}{2}$ of which is 54. Of so many units is the other right-angled triangle. ¹⁵ Altogether it is 96 units. And the figure will be as follows.

¹⁶ If there be given a rhomb such as the one drawn below according to the conditions of the problem the 10¹⁷ squared is 100 and $\frac{1}{2}$ of the 12 of the base is 6. (This) squared is 36. ¹⁸ Subtract the 36. 64 remains, the square root of which is 8. So great is the perpendicular. ¹⁹ This by the (6) of the base is 48, $\frac{1}{2}$ of which is 24. Of so many ²⁰ units is each of the right-angled triangles. Altogether 96 units. ²¹ And the figure will be as follows.

A few words should be said about the forms of the individual letters, and the symbols and abbreviations used.

A has the cross stroke horizontal, as in the Imprecation of Artemisia,¹ as against the rounded shape Δ of the Arsinoe deed² or the acute shape Λ which appears generally in the Homer papyrus.³ It must be added, however, that a form approaching the rounded shape seems to occur in col. I, process 2, line 2, but the line is fragmentary, and the writing there seems to have been crowded.

B The loops are separate, hardly touching each other. The upper loop is generally smaller than the lower.

Γ is of the plainest uncial type.

Δ has the right side often slightly prolonged above the left. The left angle is slightly the most acute. The letter somewhat resembles the Δ of the Eudoxus papyrus, as given by Blass,⁴ as well as that of the Imprecation of Artemisia.⁵

E The body of E is a curve, as in most uncials. The horizontal usually fails to touch the curve, and is not exaggerated, the upper curve projecting further to the right than the lower or the horizontal.

Z is broader than high, the lower horizontal being slightly curved.

H The ends of the second vertical often curve outward. The horizontal is rather above the middle of the letter, the horizontal and the second vertical being in some cases made without removing the pen from the papyrus. The letter resembles the H of Eudoxus.

Θ The curve varies; sometimes round, sometimes elliptical, sometimes ovoid. The horizontal stroke rarely touches either side, and is not above the middle of the letter. It seems less

¹ Pal. Soc., Series II 141.

² P. S. II 146.

³ P. S. II 64.

⁴ Chart I, Müller's Hdbuch der kl. Alt., I, p. 304.

⁵ P. S. II 141.

primitive than the θ of the Imprecation, but more primitive than the θ of the Homer papyrus and Hyperides.

K The obliques touch the vertical below the middle, the lower limb striking it at a less acute angle than the upper. The reverse is the case in the Homer papyrus and in Hyperides.

Λ Both limbs curve slightly. The left limb touches the right a little way from the latter's upper extremity, much as in Hyperides and Chrysippus.¹

Μ The middle wedge reaches further down than in the Arsinoe deed, but not so far as in Hyperides. It is much as in the Homer papyrus and Chrysippus.

N is not particularly distinctive. It resembles the N of Hyperides and the Homer papyrus.

Ξ The horizontals are connected. The middle horizontal is often somewhat shorter than the others. The ξ is ordinarily not larger than other letters. It is somewhat as in the Berlin Euripides parchment.¹

ο is sometimes small, and usually approximately round.

Π The second upright is curved as in the H. The horizontal projects somewhat over both verticals. The letter is much as in Eudoxus. A combination of Π and \circ occurs in col. III, l. 1, the second vertical of Π coinciding with the left side of the \circ .

P The loop is small, about the size of the smaller \circ . The vertical is short, the whole not larger than other letters, the vertical stopping at the line. A slight horizontal stroke is visible below the loop, as in Eudoxus.

Σ The curve is fairly even, suggesting an incomplete circle, the upper end being continued further than the lower.

T The horizontal is slightly longer than the vertical; much as in Eudoxus.

Υ The ν is broad, curved rather than angular, and touches the stem about midway of its right limb. Below the stem is sometimes deflected to the left. Otherwise it resembles the ν of Chrysippus and Eudoxus.

Φ The vertical is long, extending well above and below the extremities of the average letters. The loop is small, about the size of the small \circ . The vertical passes a little to the right of the middle of the loop.

¹ As given by Blass, Müller's H'dbuch, I, p. 304.

x The limbs are short, crossing at right angles. The letter is much like the x of Chrysippus and Eudoxus.

Ω is much as in the Hyperides papyrus.

As numerals the letters retain these forms, and are unaccompanied by any special distinguishing mark. The fraction $\frac{1}{14}$ once occurs, being written τὸ ι'Δ'. San does not occur. The other purely numeral characters are found. Koppa appears in the form ς. Vau is like the ζ of the alphabet just given, having the upper curve extended beyond the lower, and lacking the characteristic hook.

Three abbreviations occur in the text. AY for αὐτά appears three times: in column II, line 4; III 2 and 3. In each case it represents the contracted reflexive, and follows the preposition ἐφ', there being an ellipsis of the πολυπλασιαζόμενα or πολυπλασίαζε, which, in Heron for example, often precedes such expressions. APOY is found for ἀρουρῶν or ἀρουραι nine times in the text, appearing in each process at least once. ΠAΕY for πλευρά appears once, II 8. A fourth abbreviation, ΣXH for σχῆμα, occurs in one of the broken lines of column II; and a fifth, TOIOY for τοιοῦτο, immediately follows.

Five symbols are employed in the fragment. J for γίνεται or γίνονται is used twenty-seven times in the text, while it appears in connection with the figures, like APÖ above. In most of these instances it introduces a product; in a few, notably in connection with the figures, a sum.¹ The same abbreviation occurs in the Petrie Papyri, e. g. Part II, plate 13, where it is found in a taxing account. In that connection it has been explained by Professor Mahaffy as a conventionalized form of the cursive initial γ of γίνεται. The symbol would thus be in origin an abbreviation.¹

The symbol ζ I have been unable to find anywhere. I could think of nothing better for it than λαβέ, making with ἀπό, which in all four cases precedes it, the word ἀπολαβε 'subtract.' Some such value the context certainly requires; but it was objected by several scholars that the fragment's expression for 'subtract' was ἄφελε, and that it was improbable that an additional word for

¹ Cf. also Petrie Papyri, Part II, pp. 39, 73; and Baillet on the Akhmîm papyrus (Mémoires, Mission archéologique française au Caire, tome neuvième, 'Le papyrus mathématique d'Akhmîm'), where γι ordinarily appears for γίνεται; e. g. tome IX, I, p. 63, No. 1, ll. 3, 4; p. 64, No. 2, ll. 2, 3.

precisely the same purpose should be employed. It may be replied to this that Heron, for example, has three or four expressions for 'subtract,' using λαβὲ ἐξ w. gen., ἀρον ἀπό w. gen., and ἀφαίρει, beside the usual ἀφέλε; while a fifth word, ὑφέλε, appears in the Akhmîm fragment. A more serious objection is the fact that ἀπόλαβε nowhere, as far as my examination of the Greek mathematicians goes, is employed exactly as this context would require. For what seems a valuable suggestion here I am indebted to M. Heiberg, who suggests τοῦτων. This fits well with the context, except in the third line of the last process, and comports much better with the general usage of the processes. M. Tannery also, and quite independently, has proposed this reading, though without giving it his unqualified approval. The suggestions of M. Tannery and M. Heiberg were made on the basis of a provisional transcription of the process in column II. The problem presented by the occurrence of the abbreviation in the last process was not before them. Even here, however, ἀπὸ τοῦτων may stand; but only as a set phrase, used regardless of preceding context, to introduce a subtrahend.

The symbol \odot as a remainder sign occurs in almost the same form in the Petrie Papyri, the dot being lacking.¹ This might be resolved as λοιπόν, λοιπά, in the manner of Heron; hardly as λιπών, λείψει, in that of Diophantus; although it seems not impossible that M. Tannery's Λ and the \odot ² of our fragment are kindred developments.³ The Akhmîm papyrus employs λείπεται quite unmistakably, although in an abbreviated form; e. g. ἀπὸ τῶν ΣΚΑ ὕφ[ε]λ(ε) ΙΖ λ(ει)π(ε)ται ΣΔ.⁴ This symbol appears eleven times in the fragment. It has been suggested that the curved form of it connects it with Π rather than with Δ; and that it represents περίεστι rather than λείπεται. To this the subscript σ—for such it seems—presents an obstacle; and as this subscript is more uniform in the symbol's occurrences than is the curve of the symbol, it seems at least possible that we have here the ΔΟ of λοιπά or λοιπόν. I am so fortunate as to have for this opinion the confirmation of M. Tannery, whose observations on the first complete process reached me, through Professor Beman, immediately after I had written the above note.

¹ Cf. Petrie Papyri, Part II, p. 130, plate 13, bis; also p. 39.

² On \odot and Λ cf. T. L. Heath, Diophantos of Alexandria, pp. 71–3.

³ Cf. Diophantus, ed. Tannery, vol. II, proleg., pp. 35, 36, 41.

⁴ Cf. Baillet, Mémoires mis. arch. fr., tome IX, 1, p. 70, No. 13, l. 6, etc.

∟ for ἡμῖν appears eight times, being ordinarily preceded by τὸ and followed by the symbol for γίνεται. It is in the ordinary form as it appears, e. g., in the Eudoxus papyrus.¹ ∟ for τέταρτον I have observed in many second-century tax bills, and in the Akhmîm papyrus. In the Eudoxus papyrus² it occurs, and it is further twice restored by De Presle in col. 4, ll. 11, 18 of the Eudoxus. The sign appears but once in the fragment.

Professor Beman has suggested the possible connection of the fragment with the school of Heron Alexandrinus. There can be no doubt that in many of its forms of expression the fragment presents a striking similarity to the Geometry of Heron. Expressions like ὦν τὸ ἡμῖν γίνεται MH', p. 88, l. 20³; τὰ IE' ἐφ' ἑαυτὰ γίνονται ΣΚΕ', p. 86, l. 16; τοσούτων σχοινίων ἔσται ἐκάστη πλευρὰ τοῦ τετραγώνου, p. 74, l. 25; λαβὲ τὸ ἡμῖν τῆς βάσεως, p. 62, l. 10; or the following: τὰ I' τῆς βάσεως ἐπὶ τὰ IB' τῆς πρὸς ὀρθὰς γίνονται PK'. ὦν τὸ ἡμῖν γίνεται ἐξήκοντα· τοσούτων σχοινίων ἔσται τὸ ἐμβαδόν, p. 58, ll. 4-6, find rather close parallels in the fragment. A more striking matter is the resemblance in the use of certain mathematical terms. Κορυφή has in Heron the characteristically Egyptian meaning 'upper side' of a four-sided figure. Heron's definition of it reads: κορυφή δὲ ἐστὶν ἡ ἐπὶ τῇ βάσει ἐπιτιθεμένη εὐθεία, p. 44, l. 17, and in going on to define σκέλη he says: σκέλη δὲ αἱ ἀπὸ τῶν ἄκρων τῆς κορυφῆς ἐπὶ τὰ ἄκρα τῆς βάσεως τεταμέναι εὐθεῖαι, p. 44, ll. 18, 19. The expression τραπέζιον σκαληρόν, found in Heron, p. 21, l. 17, and in our fragment, is not a common one. The character of the processes, however, supplies the closest link between them. Heron concerns himself with specific problems, not with general derivations of formulas or theorems, although constantly assuming as familiar and fixed, the results of such derivations. This will be seen to correspond exactly to the material of the fragment. Another more striking similarity in method is in connection with finding the areas of irregular rectilinear figures. Heron's way is to cut a complicated figure into triangles and rectangles, and then compute the areas of these. The first figure in this papyrus, too, it will be observed, is an isosceles trapezoid, a figure for which

¹ Notices et Extraits, Planche I, col. 4, l. 23; cf. also Gardthausen, p. 268.

² N. et E., Planche I, col. 3, l. 34; col. 4, l. 15.

³ The references are to Heron's Geometry, ed. Hultsch, Heronis Alexandrini Geometricorum et Stereometricorum Reliquiae, Berlin, 1864.

Heron has been said to have a special fondness. In our second process, also, the writer's first auxiliary line seems to have reduced his figure to an obtuse-angled triangle and an isosceles trapezoid.

To this view, however, there are one or two serious objections. Perhaps no great importance attaches to the fact that we cannot in Heron find such phrases as *ὡς δεῖ, εἰς τὸ αὐτό*, for which latter Heron's equivalent is uniformly *ὁμοῦ*; and to the entire absence from Heron's measures of *ἄρουρα*. More significant must be considered the inconsistency between Heron's *παρὰλληλόγραμμον* and that of the fragment. Heron's parallelogram does not differ from Euclid's: *ἐπὶ δὲ τῶν τετραπλεύρων ἃ μὲν καλεῖται παρὰλληλόγραμμα, . . . παρὰλληλόγραμμα μὲν οὖν τὰ τὰς ἀπεναντίον πλευρὰς παρὰλλήλους ἔχοντα*, etc., p. 20, ll. 11-13. There appears to be no way of bringing the 'parallelogram' of column III of our fragment within the scope of this definition; and we are confronted with an inconsistency as remarkable as was the agreement in the case of *κορυφή*. Of Heron's method of calculating the area of a triangle in terms of its sides, our processes show hardly a trace. It has been suggested that our fragment reflects the methods of the second book of Euclid rather than the formula of Heron; but the difficulty of relating our surveyor with Euclidean terminology and method in general, has already been shown to be considerable. The Heronian formula, on the other hand, he clearly had not. Perhaps it is not much to say that he had the materials out of which that formula was derived; but I have thought it not impossible that we have in this fragment one of those early mathematical works of whose materials Heron later became the organizer and compiler; in other words, the work of which this papyrus was a copy, if not itself one of Heron's sources, may fairly represent the character of the sources he had and used.

Slight resemblances to other mathematicians may be noted. The fragment's use of *ὑπογεγραμμένον* is somewhat closely paralleled in Apollodorus, *ἔστι δὲ τὰ ὑπογεγραμμένα σχήματα*,¹ and in Bito, *τὸ δὲ σχῆμα οἷόν ἐστιν ὑπογέγραπται*²; also in a passage ascribed to Heron Byzantinus,³ *καὶ τὸ σχῆμα ὑπογέγραπται*. As to *ὡς δεῖ*, M. Tannery has suggested that it may be a scribal error for *ὡς ἀεί*, in uncials *δ* and *α* being easily confused. *Ὡς δεῖ*, however, does not seem difficult when compared with Euclid's *ὡς ἔτυχεν, ὁ ἔτυχεν, ἃ ἔτυχεν*,

¹ Before fig. 47, Wescher's *Poliorcétique des Grecs*.

² Before figs. 17, 19, 20, *ibid*.

³ Before fig. 103.

with which expressions it would stand in almost direct contrast, meaning 'by construction,' or perhaps better, 'by the conditions of the problem.'

To facilitate reference to the fragment, the following index has been prepared. Occurrences of *καί*, *δέ* and the forms of the article are not noticed in this index; nor is the material afforded from the fragments of col. I and of the last process of col. II.

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